REQUIREMENTS FOR A NEW GUIDANCE LAW AGAINST MANEUVERING TACTICAL BALLISTIC MISSILES

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Abstract

Analysis and extensive simulation studies both indicate that *classical* guidance methods cannot guarantee the terminal accuracy required for a "hit-to-kill" against maneuvering tactical ballistic missiles, which are to be expected in the future. This paper addresses the need for the development of a new guidance law of improved performance and outlines the requirements for it. The new guidance law has to be robust with respect to the type of target maneuver and has to account explicitly for the inherent limitations of the available "optimized" estimation technology. Both deterministic and stochastic requirement formulations are presented.

Introduction

All anti-ballistic defense systems, which are currently in development, have been designed to use *classical* guidance methods against non maneuvering targets. Recent flight test results [1, 2] demonstrated that state of art technology allows to intercept such targets with point-capture accuracy, validating the "hit-to-kill" concept. It seems that these anti-ballistic defense systems have already succeeded to provide some counter-proliferation incentives against existing, predictable, non maneuvering threats. In spite of that, the evolving nature of future threats has created an increasing concern of the difficulties in intercepting (yet unknown) maneuvering reentry vehicles. This concern is clearly reflected in some of the works discussed in recent TMD and BMDO conferences, as well as by the explicit solicitation of this topic in the call for papers of the present meeting.

Currently known tactical ballistic missiles (TBM) are not designed to maneuver, but they reenter the atmosphere at very high speeds. Thus, they already have an inherent maneuvering potential, comparable to that of the interceptors. Moreover, there are simple well known devices that can generate a non-zero trimmed angle of attack and consequently a very high load factor during the reentry phase. Note, that the hopefully successful development and deployment of new anti-ballistic missile defense systems (PAC-3, Arrow, etc.) may paradoxically motivate the development efforts for a generation of maneuverable TBMs.

Several studies considered periodical maneuvers of rolling airframes [3-5]. These seem to be very effective evasive maneuvers, but do not represent the *optimal* evasion in the theoretical sense (the "worst case" for the interceptor). In the course of a multi-year investigation, reported in several recent papers [6-11], the *optimal* evasive strategy of a TBM was identified as a sequence of randomly selected changes in the direction of the terminal "hard" maneuver.

Both theory and extensive simulation studies indicate that *classical* guidance methods cannot guarantee the terminal homing accuracy which is required for a "hit-to-kill" performance against a highly maneuvering TBM. For future anti-ballistic defense scenarios, where maneuvering TBMs are expected, a new guidance concept is needed. It is disturbing to observe that, while the technology of guidance sensors and of other elements involved in a missile system made a very impressive progress in the last decades, missile guidance laws remained conservative. The objective of this paper is to outline the approach for the development of a new guidance law against TBMs of high maneuvering potential and to formulate the requirements for such a guidance law .

In the next section of the paper the question: "Why *classical* guidance methods are unable to guarantee a *point capture* accuracy against highly maneuvering targets?" will be elaborated. Recent investigation results will illustrate why the TBM interception scenario has to be formulated as a zero-sum pursuit-evasion game, rather than an optimal control problem. The benefit of including the estimated target acceleration in the guidance law is discussed and the performance degradation due to the inherent estimation delay is demonstrated. Based on these results three requirements for new guidance law development are formulated.

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Problem statement

Guidance theory (assuming point-mass vehicle models and perfect information) points out that non-zero miss distances are created by three main error sources: (i) non ideal dynamics of the guidance system, (ii) the contribution of target maneuvers, (iii) limited missile maneuverability. *Modern* guidance laws, developed by applying optimal control theory [12], have included the first two effects in the *generalized zero effort miss distance* and used a time varying gain schedule. Compensation of own dynamics is a rather straight forward control task (though it requires a high bandwidth system), but for the contribution of the target maneuvers their future evolution must be known [13]. In most cases a constant target maneuver has been assumed. If the assumption on the target behavior is correct and the lateral acceleration of the interceptor does not saturate, such a guidance law (denoted in this paper as **OGL**) can reduce the miss distance to zero. The inevitable saturation is not a major concern, because if the interceptor/target maneuver ratio is sufficiently high, it occurs only very near to the end and the resulting miss distance becomes negligibly small.

However, if the assumption on the target behavior is wrong, for example the direction of the maneuver is changed near to the end, very large miss distances are created, as it is shown in Fig. 1. This figure displays the normalized miss distance (the actual miss distance divided by the product of the maximum target acceleration and the square of the missile's time constant) as the function of the normalized time-to-go (time-to-go divided by the missile's time constant) of the change in the direction of the target maneuver. The results are presented for three values of interceptor/target maneuver ratios and for a target with ideal dynamics. Note, that these results are still optimistic, because they are based on the assumption that the change in the target maneuver direction is instantaneously observed and included in the guidance law. Since estimation of unknown target maneuvers cannot be obtained without an inherent delay an additional deterioration of the homing accuracy has to be expected.

The above results demonstrate the basic deficiency in formulating the interception of a maneuverable target as an *optimal control* problem. Target maneuvers are independently controlled and as a consequence they cannot be predicted. Thus, the *optimal control* formulation is not appropriate. The scenario of intercepting a maneuverable target, having the feature of being controlled by two independent agents, has to be formulated as a *zero-sum pursuit-evasion game*. The game solution provides simultaneously the missiles guidance law (the *optimal pursuer strategy*), the "worst" target maneuver (the *optimal evader strategy*) and the resulting guaranteed miss distance (the *value* of the game).

Although the concept of such a formulation, was already published in 1965 [14], the missile community has failed to recognize the potential involved in it. While a linear quadratic game formulation with an ideal dynamic model leads to Proportional Navigation [15] as an *optimal* guidance law, a more realistic analysis has to recognize that the controls are bounded and missile dynamics has to be represented at least by a first-order transfer function with the time constant τ_p. Such an analysis [16], limited to a planar scenario, was published in 1979 and was extended later in other papers [17 - 19]. It yielded a *game optimal* guidance law (denoted in the sequel as **DGL/0**) which explicitly accounts for the limited interceptor maneuverability, allows ideal target maneuver dynamics and eliminates the need of knowing the actual target maneuver. This guidance law provides a **robustness** with respect of the type of target maneuver. The normalized *guaranteed* miss distance is a non linear function of the interceptor/target maneuverability ratio "μ". For sufficiently high maneuverability ratios ($\mu > 2$) small miss distances are guaranteed. The "worst" target maneuver is a single "hard" turn for a duration of "θ_s" (also a function of μ), starting near to the end of the interception.

If the target does not perform the optimal evasive maneuver, the actual miss distance becomes smaller than its *guaranteed value*. In Fig. 2 the guidance performances of **OGL** and **DGL/0** are compared against a maneuvering target with ideal dynamics that changes the direction of the maneuver shortly before the end (as in Fig. 1). The advantage of the robust **DGL/0** is clearly seen. Non ideal target dynamics (e. g. a first-order transfer function with the time constant τ_e) also modify the *guaranteed* miss distances, as can be seen in Fig. 3, where the target/missile time constant ratio (τ_e/τ_p) is denoted by ε.

If in this perfect information zero-sum pursuit-evasion game formulation non ideal (first-order) target maneuver dynamics is assumed and the actual target maneuver is available, an improved guidance law (denoted in the sequel as **DGL/1**), can be used [18]. This guidance law guarantees a

zero miss distance (the sufficient condition for a "hit-to-kill") if the acceleration rate (the maximum acceleration devided by the time constant) of the interceptor is superior to that of the target. This condition is expressed by the inequality $\mu\varepsilon \geq 1$, as can be seen in Fig. 4.

Unfortunately, target maneuvers cannot be either predicted or directly measured. They have to be estimated, based on the noisy measurements of the relative missile/target position. Moreover, the estimation process is never instantaneous. The estimation accuracy, as well as the time of convergence, depends on the accuracy of the target model and on the measurement noise level. It is of common experience that, even if the accuracy and convergence of a position estimate are satisfactory, the estimated acceleration is less precise and it converges more slowly. This inherent phenomenon was approximated in a recent paper [11] by assuming that the estimation process of the target's lateral acceleration yields a perfect information outcome delayed by the amount of $\Delta\theta_e$. There is a lower bound for the value of $\Delta\theta_e$, which can be found based on generic arguments, independent of the form of the estimator [21]. The inherent delay in the estimation of target maneuver deteriorates the predicted homing accuracy of any guidance law using it, such as **OGL** and **DGL/1**, as can be seen in Fig. 5. These results clearly show that if the estimation is ideal ($\Delta\theta_e = 0$), both guidance laws have a better performance than **DGL/0** - which does not use information on the actual target maneuver. However, for estimation delays longer than some threshold value, **DGL/0** (with its robust non-zero miss distance) has to be preferred. Moreover, the substantial difference between **OGL** and **DGL/1** demonstrates again the superiority of a guidance law derived using pursuit-evasion game theory.

The scenario of intercepting a maneuvering TBM is in effect an *imperfect information* pursuit-evasion game with a state constraint imposed on the *evader* (the TBM). The TBM has no information on the interceptor (the *pursuer*) and it also has to satisfy its initial objective of hitting a designated surface target. It has to be noted, that the deterministic results, which were presented earlier, belong to a *perfect information* scenario, the "worst" case for the anti-ballistic missile defense. Based on these results the main reasons for the failure of current anti-ballistic defense systems to *guarantee* a "hit-to-kill" in the interception of a highly maneuvering TBM, can be summarized by the following:

1. In order to obtain zero or negligibly small miss distances a high interceptor/target maneuver ratio ($\mu > 2$) is required. This capability cannot be uniformly guaranteed against the class of feasible reentry vehicle maneuvers expected in future anti-ballistic defense scenarios.

2. The homing accuracy of a guided missile is limited by the estimation error of the guidance system. Fast and accurate estimation of arbitrarily maneuvering targets is still an unsolved challenge.

Nevertheless, a successful interception can still be achieved if the interceptor missile is equipped with a warhead, or (using a new more "politically correct" expression) *lethality enhancer*, with a larger lethal range than the *guaranteed miss distance*.

Stochastic analysis

The deterministic analysis presented in the previous section imbedded the assumption that the TBM can perform the *optimal* evasive maneuver. In fact the TBM has no information on the interceptor missile and consequently it cannot perform such a maneuver in the deterministic sense. Therefore in most cases the actual *miss distance* will be smaller than the one predicted by the perfect information analysis. This topic is elaborated in this section summarizing result detailed in previous papers [6-11]. It is assumed from the outset that the parameters of the scenario are such that the perfect information game solution (the "worst case" from the point of view of the defense) predicts that the guaranteed miss distance is larger than the lethal range of the interceptor warhead. If it is not so, the subsequent analysis is unnecessary. Let the ratio of the lethal range and the guaranteed miss distance be denoted in the sequel by $\eta < 1$.

In order to understand the expected behavior of a future highly maneuverable TBM, the point of view of the TBM designer must be considered. His objective is to obtain, in spite of the lack of information, the highest probability of interception avoidance allowing the TBM to hit the designated surface target. Without maneuvering, or executing constant maneuvers, predictable trajectories are created and consequently the probability to be intercepted is very high. Without information on the interceptor missile the *optimal* evasive maneuver, which guarantees the largest achievable miss distance, cannot be executed. The best the designer can do is to preprogram a maneuver sequence for the TBM (subject to the constraint that its designated surface target should be reached). Since there is no information

on the interceptor missile, the timing of the maneuver sequence has to be *random.* Randomness is also necessary to deny predictability of the trajectory and to make the estimation task of the defense more difficult. Such an approach creates a so called *mixed strategy* (a probability distribution over a set of deterministic *pure* strategies).

The intuitive guidelines for the structure of the evasive maneuvers (based on the assumption that the TBM has perfect information on its own position with respect to the designated surface target) are the following:

a. The maneuvering sequence should cover the entire interception range of the defense system using maximum lateral acceleration.
b. The optimal sequence must have a *small* number of randomly timed commands of direction change (*switches*).
c. No single maneuver should be too long.
d. The duration of each maneuver should be of the order of θ_S.

In summary, the parameters of a *pure* evasive strategy are the *switching* distances from the designated surface target. The random selection of these parameters creates the *mixed strategy* of the TBM.

In this situation the defense system, in spite of having complete information on the TBM position, must select randomly the time for launching the interceptor missile, covering the entire feasible domain. Otherwise, the TBM designer can plan, knowing the interception range, to execute a successful deterministic *optimal evasion.* The best launch direction in a perfect information scenario is towards the predicted *collision point.* In the scenario of interest, characterized by an unsatisfactory outcome of the perfect information game for the defense and random maneuvers of the TBM, a non-zero initial condition "bias", based on a presumed continuous maneuver, may be considered. The magnitude of this "bias" and its direction can be also random variables. Thus, the defense can also apply a *mixed strategy.* The use of *mixed strategies* in missile guidance problems was investigated in the past in the context of air-to-air interceptions of maneuvering targets by a radar guided missile [22, 23].

One may assume that the *random* maneuver sequence of the TBM, that satisfies the guidelines a.-d., is generated by a *Random Telegraph* type control, characterized by a single parameter λ (frequently used in guided missile analysis [21]). In such a stochastic process the average duration between two subsequent direction changes (*switches*) is $1/\lambda$ and the probability that there will be no *switch* during a given period of time Δt is equal to $\exp\{-\lambda\, \Delta t\}$. This probability is independent of any past event. Based on this assumption a computationally manageable solution of the imperfect information game can be obtained, as outlined in detail in a recent paper [10]. The cost function of this game is the probability of interception avoidance, to be maximized by the TBM designer and to be minimized by the interceptor. The solution is a function of several non dimensional parameters, such as μ,ε,η and $\Delta\theta_e$., introduced earlier in this paper. For any given guidance law and the parameter λ, the probability of interception avoidance can be computed by using either the analytical method of [10] or a Monte Carlo simulation. In this paper only a single example, shown in Fig. 6, is presented. It is the stochastic equivalent of Fig. 5 using the fixed values of $\eta = 0.55$ and $\lambda = 0.94$. Actually, for each set of the parameters (μ, ε, η and $\Delta\theta_e$) the *optimal* value of λ is different. Similarly, there is no guarantee that any of the guidance laws (**OGL**, **DGL/0**, biased **DGL/0** and **DGL/1**) selected for the sake of the comparison is the *optimal* one. The comparison only shows the relative merits of the guidance laws for a given set of fixed parameters. It demonstrates, also in stochastic terms, the superiority of the pursuit-evasion game formulation, as well as the strong dependence of the homing performance on the estimation process.

The insight generated by the above presented analysis, both in deterministic and stochastic terms, leads to a requirement formulation for the development of a new guidance law against highly maneuvering targets in general and for future anti-ballistic missile defense scenarios with maneuvering TBMs in particular.

New guidance law requirements

All of the currently existing missile designs have been based on the *certainty equivalence* principle, stating that the controller of a deterministic problem and the associated stochastic problem are the same. The validity of this principle was demonstrated long ago for linear systems with quadratic cost subject to white gaussian noise (LQG). This principle leads to the so called *separation theorem*, which states that the estimation and the control processes can be optimized independently. The validity of this principle in the clearly non LQG problem of a TBM interception is questionable. The *separation theorem* in this case remains valid only in the sense that the estimation process can be

optimized independently, but the guidance law has to consider the estimator dynamics and the true statistics of the disturbance [24]. Therefore, all of the following guidance law requirements are based on a pursuit-evasion game formulation of the interception scenario and also explicitly considering the *"optimal"* estimator performance.

GLR-1. *Given the predicted level of maximum TBM maneuverability, the available maximum interceptor maneuverability and the "optimal" estimator performance (based on the available sensor accuracy), find the guidance law* ***that guarantees the smallest miss distance against all feasible TBM maneuvers.***

If the outcome of this analysis does not yield a *guaranteed* point-capture or negligibly small miss distances, the "hit-to-kill" concept becomes invalid. If it is possible, the missile should have a *lethality enhancer* that compensates for the limitation of the guidance system against a highly maneuvering target. In this case the requirement for a new guidance law against maneuvering TBMs has to be rephrased.

GLR-2. *Given the predicted level of maximum TBM maneuverability, the available maximum interceptor maneuverability and the "optimal" estimator performance (based on the available sensor accuracy), find the guidance law* ***that requires the smallest lethal radius of the warhead*** *(minimizing the weight of the kill vehicle)* ***for a guaranteed satisfactory single shot kill probability (SSKP) against all feasible TBM maneuvers.***

If the *guaranteed* satisfactory SSKP is 100% (assuming ideal reliability), the outcome remains identical. For lower levels of satisfactory *guaranteed* SSKP a detailed stochastic analysis, based on the probability distribution of the miss distances, has to be incorporated in the optimization process of the guidance law.

Assuming that the TBM maneuvers are always *game optimal* is a very pessimistic and conservative approach. The new guidance law should take advantage of the fact that the TBM has no information on the interceptor and therefore it must maneuver randomly. This approach formulates the anti-ballistic defense scenario as an *imperfect information* differential game and suggests a *mixed* (random) guidance strategy based on the following requirement.

GLR-3. *Given the predicted level of maximum TBM maneuverability, the available maximum interceptor maneuverability, the "optimal" estimator performance for the available sensor accuracy and the lethal radius of the warhead, find the guidance strategy* ***that guarantees the highest probability of successful interception against all feasible random TBM maneuvers.***

If this last requirement is adopted as a basis for guidance law development, the analysis is completely in the stochastic domain. For the sake of computational simplicity and transparency, the *Random Telegraph* assumption can be used. (Whether such a random maneuver structure is indeed an *optimal* one, it remains to be investigated.) Based on the *Random Telegraph* assumption the objective of the stochastic analysis is to find, for a given set on the scenario parameters, the *optimal mixed* guidance strategy and the *optimal* value of λ.

Conclusions

This paper addresses the urgent need to develop a new guidance concept for future anti-ballistic missile defense scenarios, where maneuvering tactical ballistic missiles are expected. This need is based on the results of a multi-year investigation, devoted to analyze the interception of highly maneuverable tactical ballistic missiles.

It is shown, that the interception scenario has to be formulated as an *imperfect information zero-sum pursuit-evasion* game. It is demonstrated that *classical* guidance methods, based on *optimal control theory,* cannot guarantee the terminal accuracy required for a "hit-to-kill" homing performance against highly maneuvering TBMs. One of the reasons of this failure is the deterioration of the homing accuracy resulting from the inherent delay in estimating the actual TBM maneuvers.

The paper outlines the requirements for the development of a new guidance law of improved performance, based on a *zero-sum pursuit-evasion* game formulation and on an explicit consideration of the *"optimal"* performance of the estimator. Three different formulations of the requirement are presented. Each of these requirement formulations calls for an advanced and innovative effort in the analysis of missile guidance, a necessary complement of the great progress accomplished in missile technology. Such an effort is performed now in the Faculty of Aerospace Engineering in the Technion.

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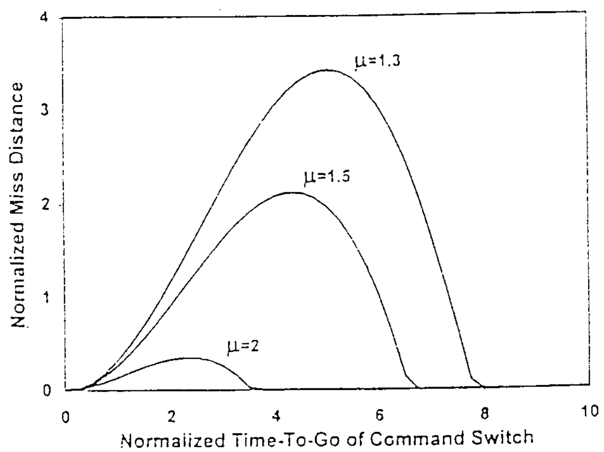

Fig. 1. OGL homing performance against a "bang-bang" target maneuver of arbitrary timing. (Ideal target dynamics.)

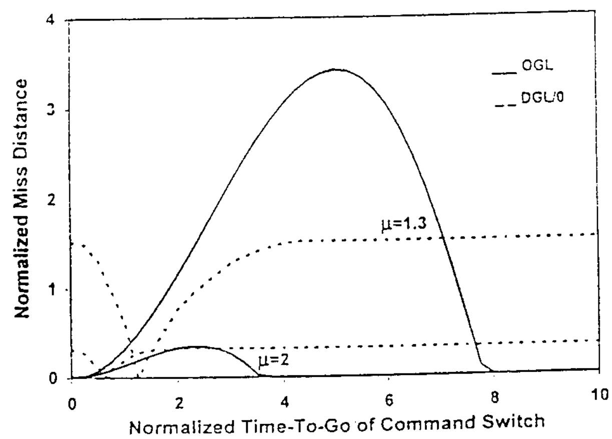

Fig. 2. Comparison of OGL and DGL/0 homing performance against a "bang-bang" target maneuver of arbitrary timing. (Ideal target dynamics.)

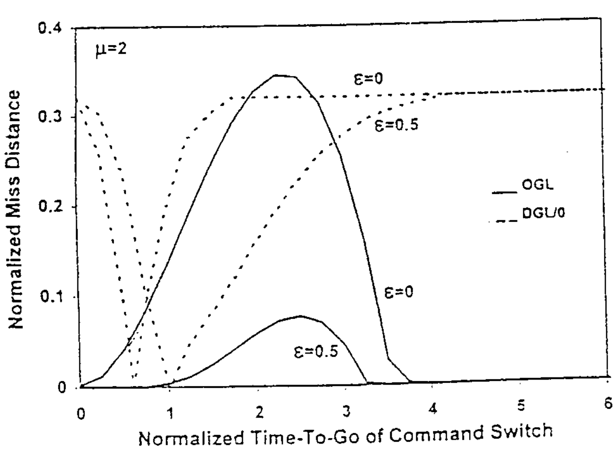

Fig. 3. The effect of non ideal target dynamics on the homing performance.

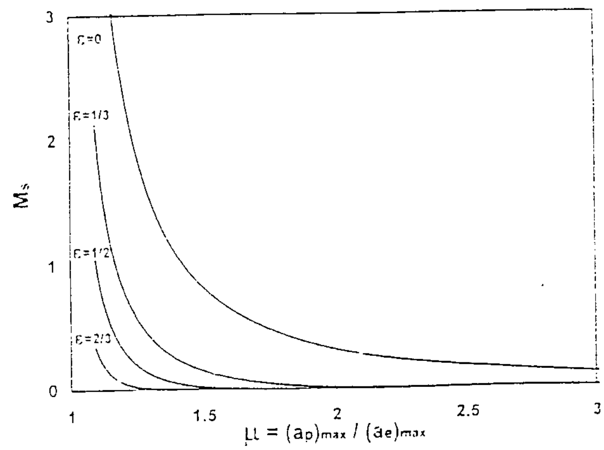

Fig. 4. Guaranteed normalized miss distance for DGL/1.

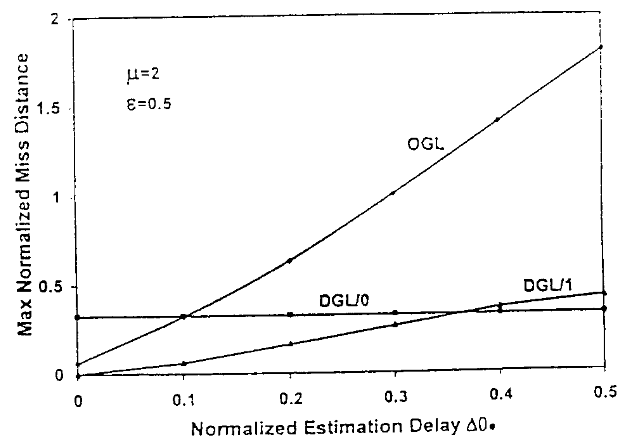

Fig. 5. The effect of estimation delay on the guaranteed miss distance.

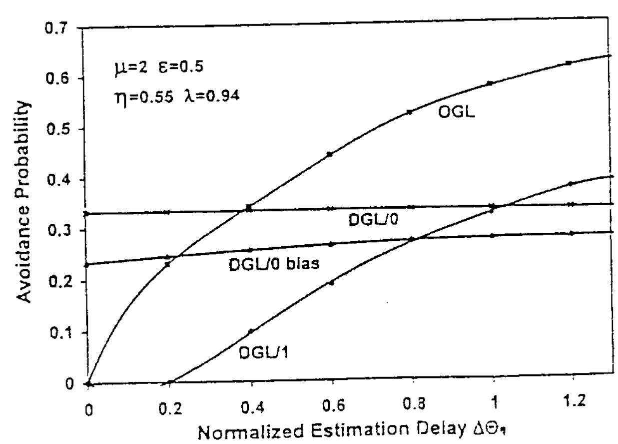

Fig. 6. The effect of estimation delay on the probability of interception avoidance.